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Solving the First Order 1-D Wave Equation: A Comparative Study of Upwind and Euler BTCS Methods

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Abstract

The simulation of wave propagation in one-dimensional media is fundamental in understanding dynamic systems governed by partial differential equations (PDEs). This study presents a comparative numerical analysis of two finite difference methods with First Upwind Differencing (explicit) and Euler Backward Time Centered Space (BTCS, implicit) will be applied to the first-order 1-D linear wave equation. The equation models the transport of a disturbance in a closed tube using a constant wave speed of 300 m/s. Both methods are implemented in MATLAB, and simulation results are analysed based on stability, accuracy, and computational characteristics. The upwind scheme demonstrates satisfactory performance with moderate accuracy but suffers from numerical dissipation at lower grid resolutions. In contrast, the Euler BTCS method shows robust stability and higher fidelity to the initial condition, even with relatively large time steps. Results confirm that both methods are capable of generating stable solutions; however, the implicit method offers greater reliability for long-term simulations. This study highlights the trade-offs between computational simplicity and numerical robustness in solving hyperbolic PDEs using finite difference schemes.

Keywords: 1-D wave equation, finite difference method, upwind differencing, Euler BTCS, MATLAB, numerical simulation

Abstrak

Simulasi penjalaran gelombang pada media satu dimensi merupakan hal yang mendasar dalam memahami sistem dinamis yang diatur oleh persamaan diferensial parsial (PDE). Studi ini menyajikan analisis numerik komparatif dari dua metode beda hingga-First Upwind Differencing (eksplisit) dan Euler Backward Time Centered Space (BTCS, implisit)-yang diterapkan pada persamaan gelombang linier orde satu 1-D. Persamaan ini memodelkan transportasi gangguan dalam tabung tertutup menggunakan kecepatan gelombang konstan 300 m/s. Kedua metode tersebut diimplementasikan dalam MATLAB, dan hasil simulasi dianalisis berdasarkan stabilitas, akurasi, dan karakteristik komputasi. Skema upwind menunjukkan kinerja yang memuaskan dengan akurasi sedang tetapi mengalami disipasi numerik pada resolusi grid yang lebih rendah. Sebaliknya, metode Euler BTCS menunjukkan stabilitas yang kuat dan ketepatan yang lebih tinggi terhadap kondisi awal, bahkan dengan langkah waktu yang relatif besar. Hasil-hasil penelitian mengkonfirmasi bahwa kedua metode tersebut mampu menghasilkan solusi yang stabil; namun, metode implisit menawarkan keandalan yang lebih besar untuk simulasi jangka panjang. Studi ini menyoroti trade-off antara kesederhanaan komputasi dan kekokohan numerik dalam menyelesaikan PDE hiperbolik dengan menggunakan skema beda hingga.

Kata kunci: 1-D wave equation, finite difference method, upwind differencing, Euler BTCS, MATLAB, numerical simulation

I. INTRODUCTION

Partial differential equations are an essential area of mathematics that is used to simulate numerous physical issues in other fields such as chemistry, engineering, and physics, among others. Analytical methods can be used to solve partial differential equations; however, not all partial differential equations can be solved analytically, necessitating the use of numerical methods [1]. The numerical method is a technique for obtaining approximate solutions to problems involving differential equations. There are several sorts of numerical methods, including finite difference methods, finite element methods, mesh methods, and spectral methods. Numerical methods provide approximate solutions to PDEs and are crucial in engineering and scientific computations.

If there are initial circumstances but no ultimate conditions, the issue is evolutionary and may be solved numerically by increasing time from the original conditions. Different numerical experts and researchers in Mathematics and related fields have used the finite difference methods a lot [2]. Among these, the wave equation plays a central role in modelling the propagation of waves, such as sound, water, or electromagnetic waves, through a given medium. Specifically, the one-dimensional (1-D) wave equation is widely used to simulate wave propagation in pipes, strings, and transmission lines due to its mathematical simplicity and physical interpretability [3], [4]. Mathematica's so-called "method of lines" is the most efficient way to achieve it. The issue is first discretised into spatial variables, and spatial derivatives are approximated using differences. This converts the PDE to a time-based system of ODEs. The resultant system of ODEs is then solved using one of the high-performance ODE solvers. PDEs and ODEs are solved using NDSolve in Mathematica.

Solving PDEs analytically is often restricted to idealized cases with simplified boundary and initial conditions. In real-world applications, especially those involving complex geometries, non-linearities, or irregular boundaries, analytical solutions are either extremely difficult or impossible to obtain [5]. Therefore, the difference finite methods (FDMs) are frequently employed among diverse numerical approaches for solving PDEs and initial and boundary problems. FDM transforms differential equations into algebraic equations by discretising the domain into a finite number of grid points. These methods are developed from the shortened Taylor's series, in which a given PDE, boundary conditions, and beginning conditions are replaced by a set of algebraic equations, which are subsequently solved using several well-known numerical techniques [6]. And the FDM method transforms differential equations into algebraic equations by discretising the domain into a finite number of grid points [7]. Because of the ease of analysis and computer codes in addressing issues with complicated geometrical structures, these approaches offer substantial benefits over other methods.

Among the various FDM schemes, the First Upwind Differencing Method and the Euler Backwards Time Centred Space (BTCS) Method are two commonly used techniques. The upwind scheme is an explicit method that accounts for the direction of wave propagation, making it suitable for hyperbolic PDEs such as the linear convection equation. However, it is often conditionally stable and may suffer from numerical diffusion. On the other hand, the Euler BTCS method is an implicit scheme known for its unconditional stability, which allows for larger time steps without compromising the reliability of the solution [6], [8].

Stability and accuracy are key considerations in choosing a numerical scheme to solve PDEs. Although explicit schemes are computationally cheaper per time step, they often require small time steps to maintain stability. Implicit methods, although more computationally demanding as they require solving the system of equations at each step, provide superior stability characteristics, especially in long-term simulations or when coarse temporal resolution is desired [6].

This study focuses on applying and comparing the First Upwind Differencing Method and the BTCS method for solving a first-order 1-D linear wave equation derived from the simplified Navier-Stokes equation, which includes only the accumulation and convection terms. The modelled system assumes a constant wave speed and closed boundary conditions, with an initial disturbance introduced in the form of a half-sinusoidal wave. Both methods are implemented in MATLAB, and the numerical results are evaluated in terms of accuracy, stability, and computational efficiency. The comparison aims to provide insights into the trade-offs between explicit and implicit schemes in simulating hyperbolic PDEs related to fluid dynamics and wave motion.

A. Problem statement

The equation given below is the most common in CFD; it kept only the accumulation and convection terms for the x component of the velocity from the Navier-Stokes equation as we already know, in CFD the variables to be computed are velocities[9]. The mathematical problem is derived from the simplified Navier-Stokes equation, retaining only the accumulation and convection terms. The resulting first-order hyperbolic PDE takes the form:

$$\frac{\partial u}{\partial t} = -a\frac{\partial u}{\partial x} \qquad a > 0$$

where u is the wave function and a is the speed of sound = 300 m/s (altitude of 10 km), assumed constant. The problem considers a closed tube with zero-velocity boundary conditions, initially disturbed by a half-sinusoidal wave in a specified region. The goal is to numerically simulate the wave propagation and assess the performance of both methods.

II. RESEARCH METHODS

A one-dimensional wave problem is employed in this report. The system in question is supposed to be a one-dimensional long tube with length L=400m. Initially, the wave on the tube received a half sinusoidal disturbance with a length of 50 to 110m. Because both ends of the tube are closed, the wave at both ends is believed to be zero. After the disruption, the system will be simulated.

A. Problem Specification

The governing equation used in the project is a one-dimensional (1D) wave equation, which is written as the equation below:

$$\frac{\partial u}{\partial t} = -a\frac{\partial u}{\partial x} \qquad a > 0$$

Where is the sound speed, which in this example is set at 300 m/s. This equation also demonstrates that the system is expected to be in a perfect environment, with no external sources of distraction [5].

B. Governing Equation

The following first-order linear partial differential equation is of the form, and the governing equation used in this report:

$$\frac{\partial u}{\partial t} = -a\frac{\partial u}{\partial x} \qquad a > 0$$

Where, is the speed of sound, which in this case is selected to be 300 m/s. This equation also shows that the system is assumed to be in an ideal environment, without any distraction from any external source.

C. Initial and Boundary Conditions

As a first step, consider the first-order wave equation,

$$\frac{\partial u}{\partial t} = -a\frac{\partial u}{\partial x}, a > 0$$

Where a, the speed of sound, is selected to 300 m/s. Assume that at time t=0, a disturbance of half sinusoidal shape has been generated. The domain represents a closed tube of length L= 400, with boundary conditions that enforce zero velocity at both ends. The initial condition is specified as, with the initial condition;

$$u(x,0) = 0 \qquad 0 \le x \le 50$$

$$u(x,0) = 100\ \{\sin\left[\pi\left(\frac{x-50}{60}\right)\right]\} \qquad 50 \le x \le 110$$

$$u(x,0) = 0 \qquad 110 \le x \le 400$$

Assuming that the disturbance is introduced in a one-dimensional long tube with both ends closed, the imposed boundary conditions are

$$x = 0 \;\; u(0,t) = 0.02$$

And

$$x = L \; u(L,t) = 0$$

According to Kreiss' theory, there are three conditions that can be determined inside, and one outside, while the other conditions are determined by the solution of the differential equation, so it is not suitable to determine the free flow conditions at the outer boundary [11].

D. Descretisation Process

To solve the equation numerically, two approaches were considered. The first upwind differentiating approach and the Euler BTCS method are the two ways. To make the problem easier to tackle, both strategies will be employed to discretise it.

i. The first upwind differencing method

The Upwind Differencing is a one-sided differencing scheme used to numerically discretise the hyperbolic partial differential equation, and the side of the differencing is determined by the direction of the flow of information. It takes into account the "flow of information" from the characteristics of the system of equations. If we consider the linear convection or the linear wave equation[12].

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -a\frac{u_i^n - u_{i-1}^n}{\Delta x}$$

Backwards differencing of the spatial derivative produces a finite difference equation of the form.

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -a\frac{u_{i+1}^n - u_i^n}{\Delta x}$$

$$u_i^{n+1} = u_i^n - a\frac{\Delta t}{\Delta x}(u_i^n - u_{i-1}^n)$$

$$u_i^{n+1} = u_i^n - c(u_i^n - u_{i-1}^n)$$

where is $c = a\frac{\Delta t}{\Delta x}$ is the Courant number, which must satisfy the stability condition $c \le 1$ for explicit methods.

ii. Euler BTCS method

The implicit formula of the Euler backwards time and control space application is unconditionally stable and is of order $[(\Delta t), (\Delta x\}^2]$. This approximation applied to the model equation yields:

$$\frac{u_i^{n+1} - u_i^n}{\Delta t} = -\frac{a}{2\Delta x}\left[u_{i-1}^{n+1} - u_{i-1}^{n+1}\right]$$

Or

$$\frac{1}{2}cu_{i-n}^{u+1} - u_i^{n+1} - \frac{1}{2}cu_{i+1}^{n+1} = -u_i^n$$

Once this equation is applied to all grid points at the unknown time level, a set of linear algebraic equations will result. Again, these equations can be represented in a matrix form, where the coefficient matrix is tridiagonal[10].

iii. Software used

MathWorks MATLAB is a multi-paradigm programming language and numerical computation environment. Matrix manipulation, visualising functions and data, algorithm implementation, developing user interfaces, and communicating with programmes written in other languages are all possible with MATLAB. The MATLAB application was used in this report to simplify and insert all equations into the program.

III. RESULTS AND DISCUSSION

Simulations were conducted to solve the one-dimensional wave equation using two numerical approaches, namely the First Upwind Differencing and Euler BTCS. Both methods are applied to a 400 meter closed tube system with a constant wave speed a = 300 m/s. The initial condition of the sine half-wave-shaped disturbance is given in the range of 50 m to 110m.

A. The First upwind differentiating method

The simulation of the First Upwind Differentiating method in this report used several parameters summarised in Table 1. Simulations using the upwind method were conducted with a space step length of 5 metres and a time step of 0.01 seconds. Numerical velocity profiles were calculated at grid points and the results were compared with the exact profiles.

TABEL I
SIMULASI PARAMETER FOR UPWIND METHOD

Parameter	Value	Description
t	0.01 s	Simulated time
L	400 m	Length of the tube
Δt	0.01 s	Time step
Δx	5 m	Length step
a	300 m/s	Speed of sound

As explained earlier, the first method that will be used to simulate the system is the upwind differencing method. The discretised equation will generate the code used to simulate the system, with the parameters that have been set. Then simulate and count in the Matlab software and produce the code that can be seen in the attachment, which is a simulation result. The study by Alam et al. (2023) emphasised the importance of error analysis and convergence in the evaluation of numerical methods for one-dimensional wave equations [13]. They performed extensive error analysis and convergence testing to ensure that the numerical solution approached the analytical solution with the expected level of accuracy.

In addition, research by Ayalew et al. (2023) developed a quadratic upwind differencing scheme in a finite volume approach to solve the convection-diffusion equation [14]. They showed that this scheme has third-order accuracy in spatial approximation and conditional stability. Although their main focus is on the convection-diffusion equation, the approach is relevant for the wave equation as both involve propagation phenomena. The study highlights that improved accuracy can be achieved by using higher order upwind differencing schemes and to develop a quadratic upwind differentiation scheme in a finite volume approach to solve this equation.

We will be looking at the convective velocity plot and we will see how the wave propagates through the domain. To visualise the accuracy of the solution, the numerical velocity profile is compared with the exact velocity profile for different numbers of grid points. There are multiple plots to examine how an increase or decrease in the number of grid points deviates the velocity profile from the exact one. As shown in Fig 1, the simulation is stable. The wave height is increasing at the same rate for both of them. In performing the simulations in this study, the use of the First Upwind Differencing method with a space step length of 5 metres and a time step of 0.01 seconds has resulted in a stable numerical velocity profile.

However, as shown in previous studies, improved accuracy and reduced numerical dissipation can be achieved by using a higher order upwind differencing scheme or a finer discretisation approach. Therefore, for simulations that require high accuracy and especially in the long term, consideration of using higher order methods or a combination of explicit and implicit methods may provide better results.

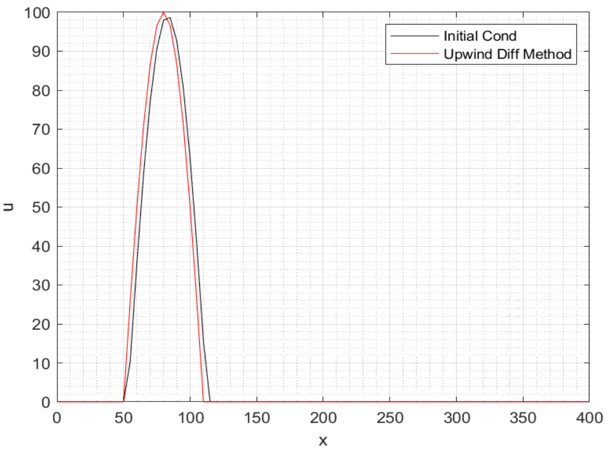

Fig 1. Simulation result of the upwind method

B. Euler BTCS Method

The simulation of the Euler BTCS method in this study used several parameters summarised in Table 2. In the Euler BTCS method, an implicit scheme is used with a space step length of 0.1 metres and a time step of 0.01 seconds. The tridiagonal matrix generated from the discretisation is solved using the direct method to obtain the velocity distribution at each grid point and time. As highlighted by Cui et al., who showed that implicit methods like BTCS preserve wave amplitude even with aggressive time-stepping [6].

TABEL II
SIMULASI PARAMETER FOR EULRT BTCS METHOD

Parameter	Value	Description
t	0.01 s	Simulated time
L	400 m	Length of the tube
Δt	0.01 s	Time step
Δx	0.1 m	Spetial step
a	300 m/s	Speed of sound

As explained earlier, the first method that will be used to simulate the system is the upwind differencing method. The discretised equation will generate the code used to simulate the system, with the parameters that have been set. Then put it into the Matlab software and produce the code that can be seen in the attachment is a simulation result. To visualise the accuracy of the solution, the numerical velocity profile is compared with the exact velocity profile for different numbers of grid points. There are multiple plots to examine how an increase or decrease in the number of grid points deviates the velocity profile from the exact one.

The second method that will be used in this report is the Euler BTCS method. The discretisation process is discussed in the chapter before. The study by Wang et al. (2021) developed an implicit finite difference scheme with high accuracy in time and space for acoustic wave propagation modelling. They showed that this method effectively suppresses temporal and spatial dispersion, and maintains high-accuracy waveforms despite using large time steps. As shown in the figure above, the simulated system is stable. The Fig 2 confirm the method's superiority in maintaining waveform fidelity, its supported by Omowo et., al. in their research [15], attributed this robustness to the implicit scheme's ability to handle high-frequency components more effectively. However, the computational cost of solving tridiagonal systems remains a trade-off [8].

In addition, the study by Morales-Hernandez et al. (2022) confirmed that the implicit scheme exhibits unconditional stability, allowing the use of larger time steps without causing solution divergence. This provides greater flexibility and efficiency in simulations compared to explicit schemes, which often require small time steps to maintain stability.

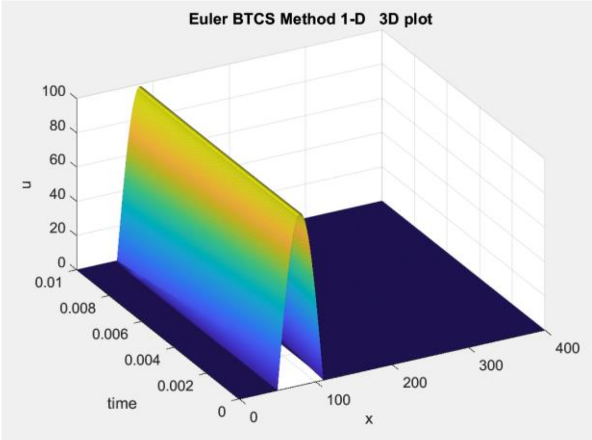

Fig 2. Simulation result for Euler BTCS method Method

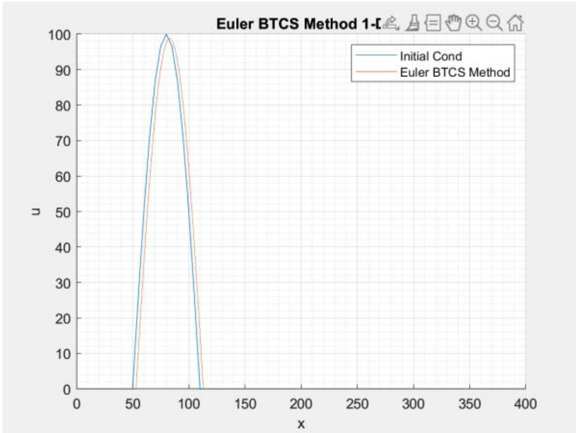

Fig 3. Simulation result for the Euler Method

As shown in the Fig 3 above, the simulated system is stable. The results obtained from the two methods used show the stability of both. This shows that the method works well, consistently and in accordance with the analytical solution. These findings reinforce the simulation results obtained in this study, showing that both the upwind and Euler BTCS methods can produce stable and accurate solutions to the one-dimensional wave equation. The choice of method depends on the specific needs of the simulation, where the upwind method offers simplicity and computational efficiency, while the Euler BTCS method provides higher stability for long-term simulations.

C. Comparison of the First Upwind Differentiating and Euler BTCS Method

The difference between the two explicit and implicit methods lies in the parameters used in these two methods. Although the parameters used in both methods are the same, the simulation results show significant differences in terms of accuracy and stability. The upwind method, as an explicit scheme, produces a simple and computationally fast solution. However, the simulation results show numerical dissipation, which causes the wave amplitude to decrease over time. Kurganov et al. (2021) further demonstrated that central-upwind schemes are susceptible to excessive numerical dissipation in convection-dominated problems unless adaptive dissipation control is employed [16]. This is common in first-order explicit schemes, especially when the number of grid points is relatively small or when the space step length is large. Despite remaining stable, this method shows limitations in maintaining the original waveform, especially in long-term simulations. Adak (2020) also highlighted that explicit methods tend to lose accuracy over extended time steps and under coarse grid conditions when compared to their implicit counterparts [17].

In contrast, the implicit Euler BTCS method shows superiority in terms of stability and accuracy. The resulting waves retain their initial shape and amplitude very well, even when small space steps and a larger number of grid points are used. The scheme is theoretically not bound by the Courant value constraint for stability, thus allowing the use of larger time steps without compromising the accuracy of the solution. Although the method is computationally more complex, as it requires solving a system of linear equations at each time iteration, this advantage is worth the quality of the results obtained. Study by Suárez-Carreño et.,al. (2021) emphasizes that BTCS methods maintain waveform fidelity across a wide range of time step sizes, especially in stiff systems where explicit methods fail to converge [18].

A comparison between explicit (Upwind) and implicit (Euler BTCS) methods in solving one-dimensional wave equations shows that both methods can provide stable results and match the theoretical expectations. However, there are some important differences between them. In terms of stability, the Euler BTCS method has a significant advantage, especially when used with large time steps, making it more suitable for long-term simulations or systems that require high accuracy. On the other hand, the Upwind method is prone to dispersion errors that can cause wave damping, especially at low grid counts, which impacts the accuracy of the results. In terms of computational efficiency, the Upwind method is simpler and faster as it does not require linear system solving, although this advantage becomes limited when the simulation scale increases or higher precision is required. In terms of implementation, both methods can be implemented relatively easily in MATLAB. However, the Euler BTCS method requires matrix construction and solving, which adds complexity to the code written.

IV. CONCLUSION

To describe the relationship between one or more elements in the universe of mathematics, partial differential equations are often used. One of the simplest versions is the wave equation, which represents the connection between one particle and another in a wave. Partial differential equation solutions, on the other hand, are not always easy to compute. As a result, several methods for solving one or more forms of partial differential equations exist. Two well-known approaches are the first upwind differentiating approach and the Richtmyer multi-step method. This study aimed to generate a wave using wave equations using those two ways.

From the analysis, it can be seen that the method used provides a good approximation solution of both implicit and explicit methods. Therefore, what we can conclude is that not only is there a limit to reducing the time step size to get the exact solution, but there is also a limit to increasing the time step size to get accurate results. For convection problems, the numerical scheme must be stable. Consistency does not necessarily mean solution stability. On the other hand, this will lead to an increase in simulation time.

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